Bayesian model selection for Markov chains using sparse probability vectors

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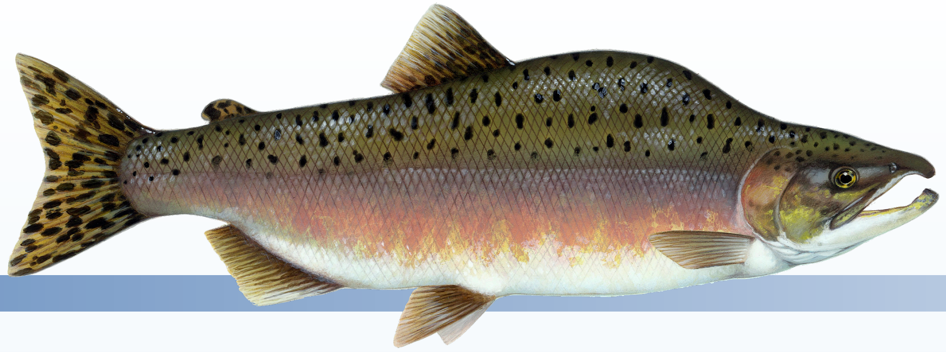
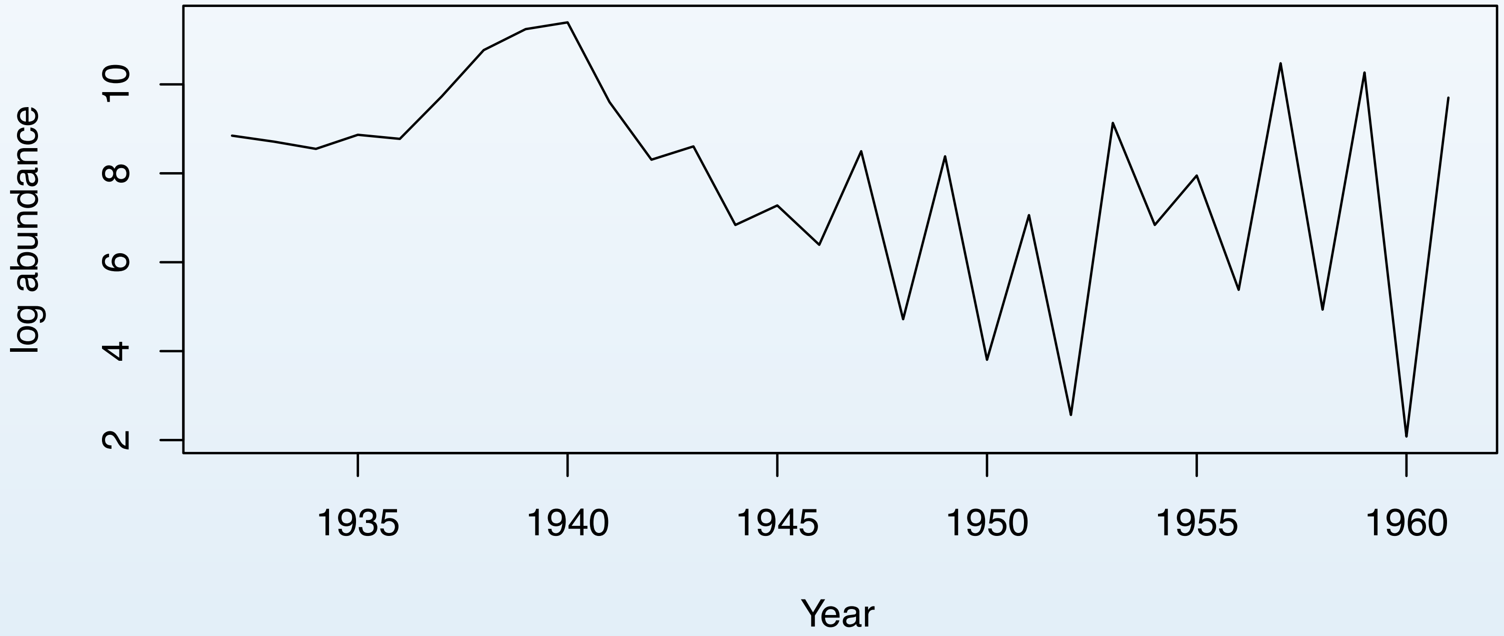
Pink Salmon time series





Data: Alaska Fisheries Science Center (2018), "AFSC/ABL: Pink salmon data collected at Sashin Creek Weir 1934-2002," NOAA National Centers for Environmental Information, https://inport.nmfs.noaa.gov/ inport/item/17256.

Image: U.S. Fish and Wildlife Service

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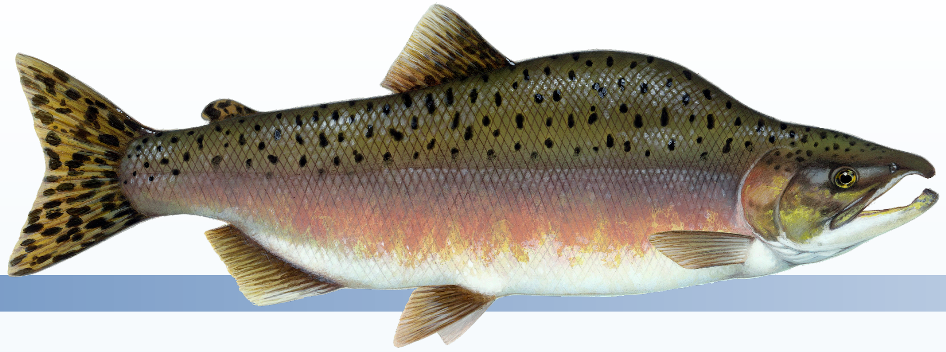

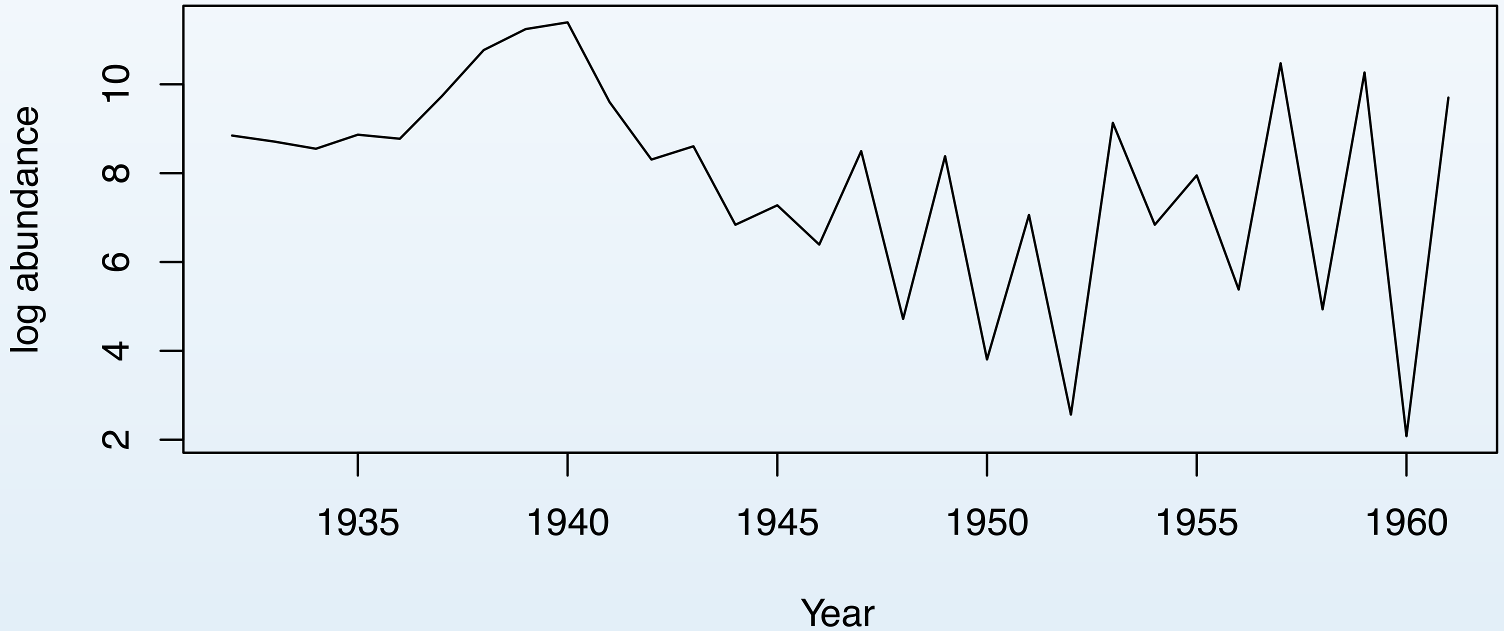

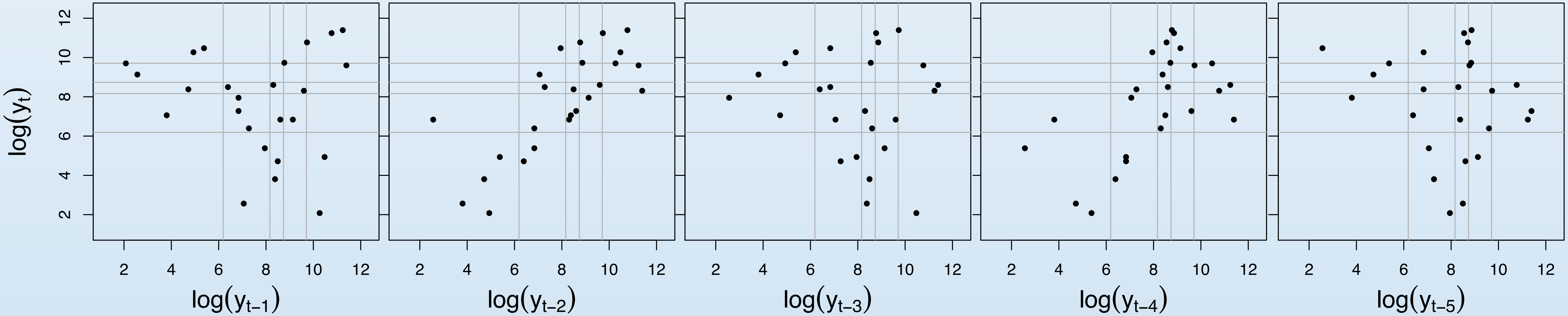

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Raftery (1985)

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Example with two states and three lags:

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first order
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Sparse probability vectors

Sparse Dirichlet mixture (SDM) prior

$$p(\boldsymbol{\theta}) \propto \mathrm{Dir}(\boldsymbol{\theta}; \boldsymbol{\alpha}) \times \sum_{k=1}^{K} \theta_k^{\beta}, \quad \beta > 1$$

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$$p(\boldsymbol{\theta} \mid \boldsymbol{n}) \quad \propto \quad \prod_{k=1}^{K} \theta_k^{n_k} \times \mathrm{Dir}(\boldsymbol{\theta}; \boldsymbol{\alpha}) \times \sum_{k=1}^{K} \theta_k^{\beta} \quad \propto \quad \mathrm{Dir}(\boldsymbol{\theta}; \boldsymbol{\alpha} + \boldsymbol{n}) \times \sum_{k=1}^{K} \theta_k^{\beta}$$

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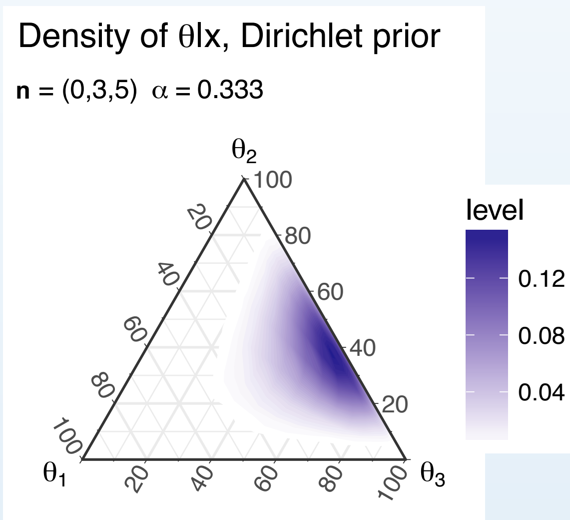

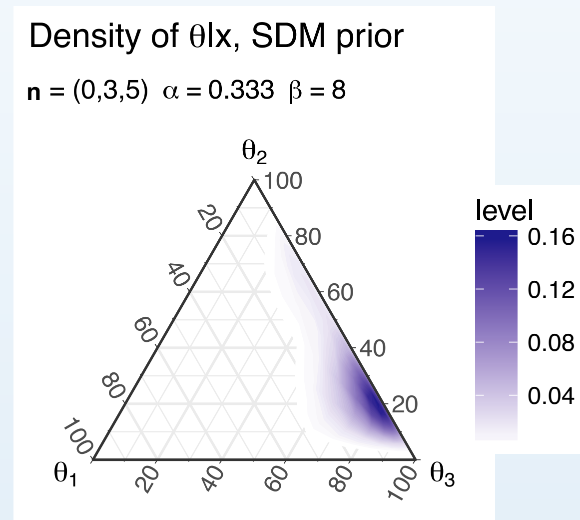

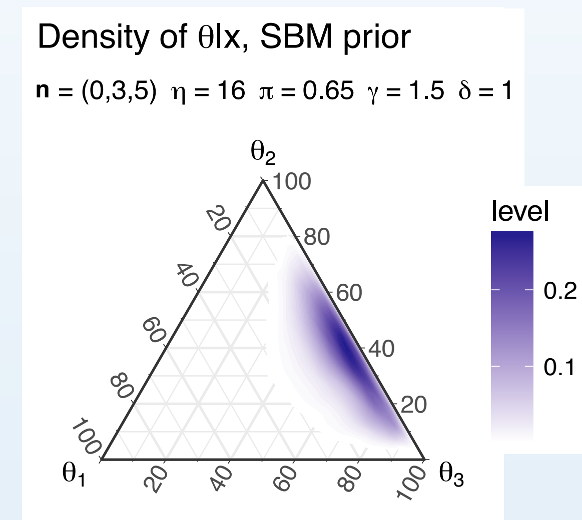

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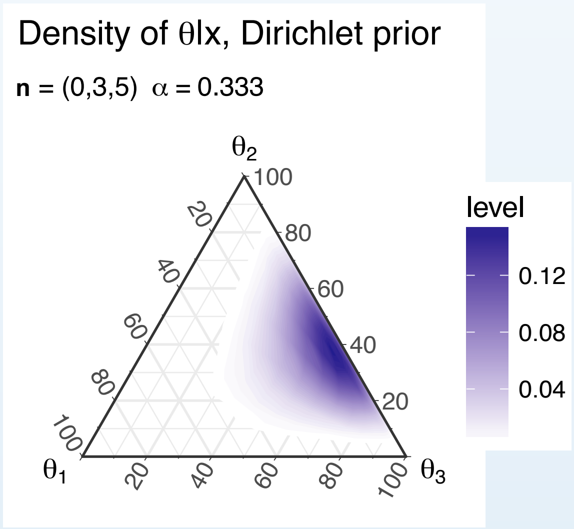

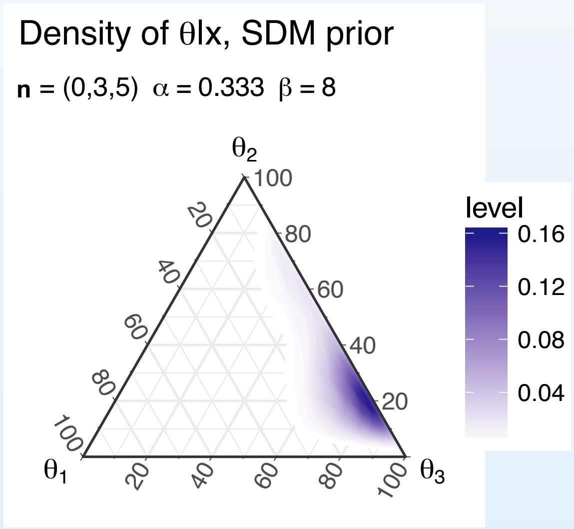

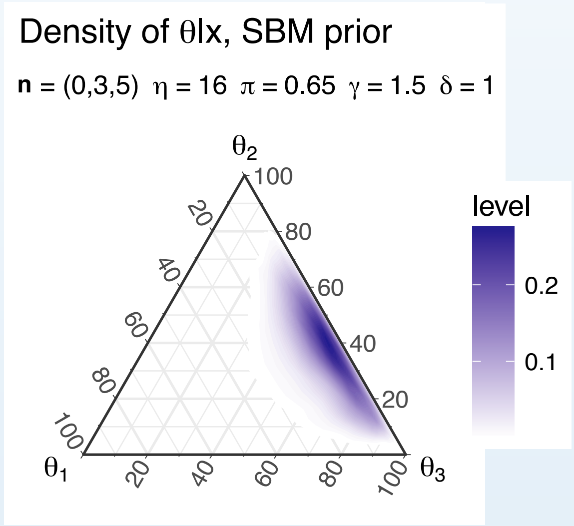

Lag inclusion inference, salmon data

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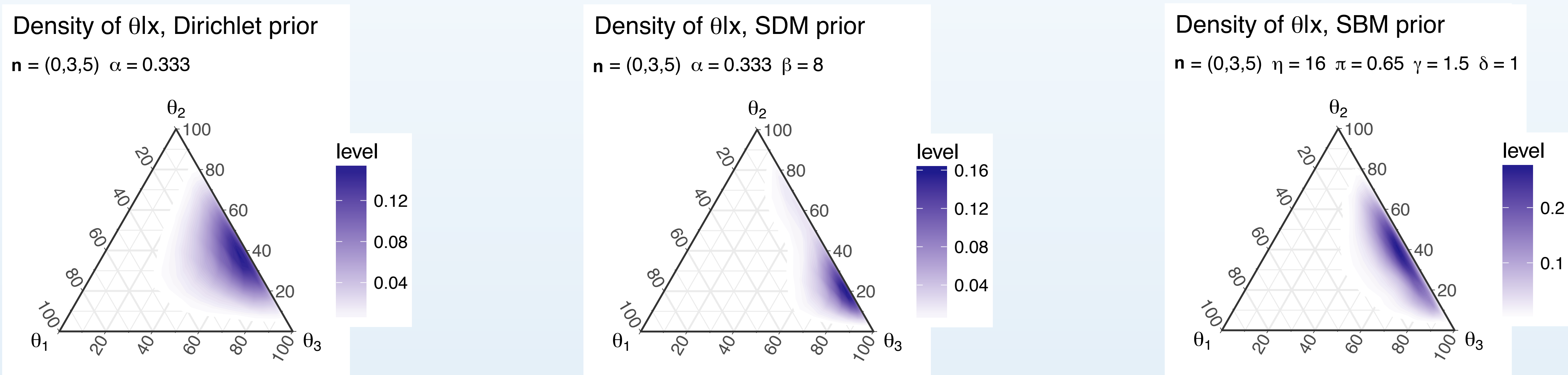

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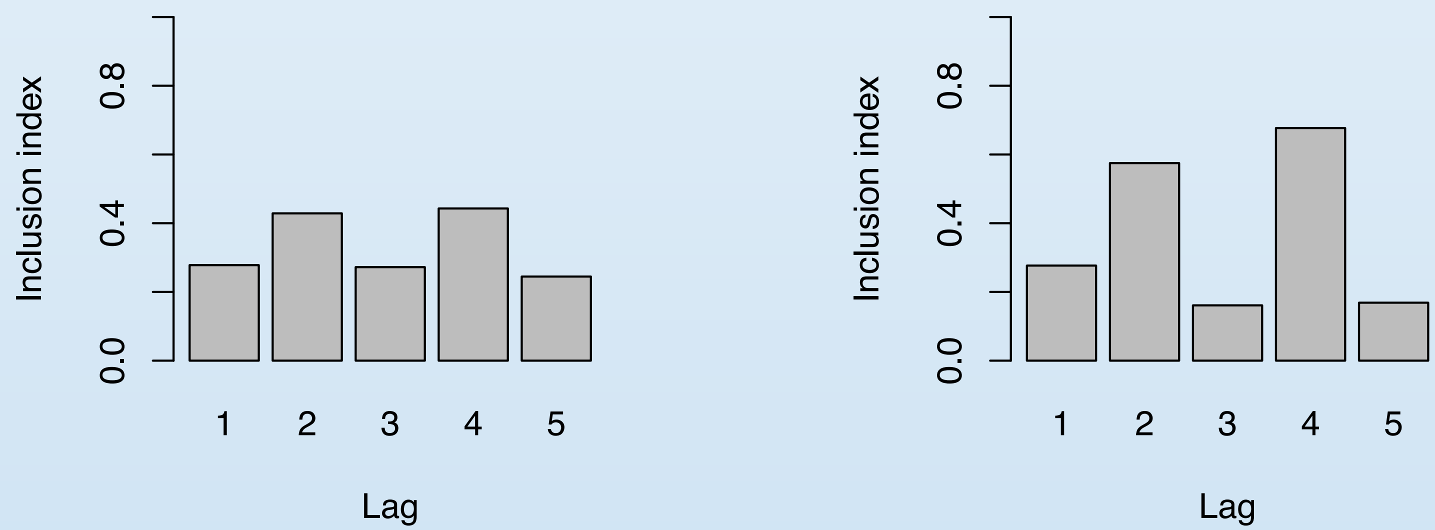

Dirichlet prior SDM prior

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